Mechanics and Mechanical Engineering
Vol. 14, No. 2 (2010) 257–268

Influence of Strain Rate on Crushing Behaviour of Thin–walled Members

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Received (13 Juny 2010)
Revised (15 July 2010)
Accepted (25 July 2010)

Results of the analysis into an influence of the strain rate upon crushing behaviour of thin–walled beams and columns are presented. The problem is solved using Finite Element Method and an analytical method based on the plastic mechanism approach. Subjects of analysis are top hat section columns under uniform axial compression and box–section beam under pure bending.

The FE analysis taking into account the strain rate effect is performed, simultaneously with the plastic mechanism analysis (yield line analysis) in order to investigate the crushing behaviour of the member. The Cowper–Symonds constitutive relation is taken into consideration. The strain - hardening effect is neglected, so that in the analytical plastic mechanism solution the rigid – perfectly plastic material is taken into consideration, while FE analysis takes the strain hardening into account.

Comparative diagrams of structural behaviour (loading paths) of thin-walled sections under investigation for different strain rates are presented. Some conclusions and remarks concerning the strain rate influence are derived.

Keywords: Thin–walled members, dynamic crushing, strain–rate sensitivity, plastic mechanism

1. Introduction

Loadcarrying capacity and energy absorption of thin–walled structures subjected to dynamic load is an important problem in two areas of engineering applications: first – energy absorption of members acting as energy absorbers during accidental collision of automotive or rail vehicles [1], secondly – load-carrying capacity of structural members subjected to seismic loads [2].

The term "dynamic crushing" is used in the present paper in the sense of progressive crushing of a structure subjected to the impulse of an applied load.

The behaviour of many structural materials, particularly mild, low–carbon steel, to less extend – aluminum alloys, display a strong influence of the strain rate upon the stress- strain characteristics. This influence can not be neglected in the analysis of thin–walled structures subjected to dynamic loads. Especially it concerns thin–walled members applied as energy absorbers.

The strength of thin–walled structures is usually calculated on the basis of "effective width" model and their ultimate capacity is evaluated using a reduced or effective crossection and, additionally, the elastic limit for maximum stress. This approach is currently used in almost all design codes. However, short thin–walled columns and beams display a significant post–elastic capacity. It means, that the actual load–carrying capacity of such a member is higher than the ultimate load calculated using the method mentioned above [2]. It concerns both static and dynamic loads.

In order to evaluate properly the "redundancy" of the load–capacity one has to solve the problem of stability and post-buckling behaviour of the structure, as well as the problem of failure behaviour (plastic mechanism problem), i.e. to estimate the upper–bound load–carrying capacity [2]. That estimation consists in the determination of the intersection–point of a post–buckling path (evaluated using either analytical method or numerical one, e.g. Finite Element Method) and a rigid–plastic *failure curve* obtained from the plastic mechanism analysis [3].

If a thin–walled member is subjected to dynamic load, particularly dynamic progressive buckling, this estimation has to count for the strain–rate influence upon stress–strain material characteristics. It is particularly important when a thin–walled member works as an energy absorber. In the presented solution we limit the considerations to the "*dynamic progressive crushing*", which means that we take into account the strain–rate but neglect inertia effects.

In the literature one can find many different approximate, empirical constitutive relations, taking into account the strain rate. Among them Cowper-Symonds [4,5] relation has been confirmed in experimental tests of different materials:

$$\bar{\sigma}_0 = \sigma_Y \left[1 + \left(\frac{\dot{\varepsilon}}{D} \right)^{1/q} \right] \tag{1}$$

where σ_Y is the initial yield stress at static load, $\bar{\sigma}_0$ - initial yield stress at dynamic load, $\dot{\varepsilon}$ - strain rate, while q and D are empirical coefficients determined from the tensile/compressive test diagram. Jones [4] quotes coefficients q and D for different materials taken from other authors.

Expression (1) gives the relation between the initial yield stress at dynamic load and the strain rate but it does not account for strain–hardening effect.

The increase of the flow stress , or the effective stress (when the strain hardening effect is additionally taken into account) plays an important part, when a designer has to have a better approximation of the energy absorption of the thin–walled member and its load–capacity under dynamic load.

In consequence, it is necessary to modify the plastic load–capacity (the plastic flow stress) in the solution of the plastic mechanism problem, in order to cater for the enhancement of the flow stress with the strain rate. The strain–rate sensitivity has been taken into the consideration by Jones [4] in the solution of the dynamic crushing of thin–walled tube subjected to axial compression. Jones replaced in the solution derived originally by Alexander, the static flow stress (initial yield stress) by the dynamic flow stress according to Cowper–Symonds constitutive relation [5].

The strain–rate sensitivity analysis has been carried out also in the case of lateral crushing of thin–walled beams by Kotelko et al. [6], where both analytical plastic mechanism approach and FE analysis has been applied.

In recent years some research has been carried out into the transition from progressive local buckling to global bending failure mode in thin–walled columns (bars) under axial impact. Such a transition is observed in the stage of collapse of relatively long bars and at impact velocities higher than a certain "critical" one. It was reported among other researchers by Kotelko and Mania [12]. Determination of "limit"' or "critical" values of bar's length and impact velocity is still an open question. Results of research devoted to this problem (both theoretical and experimental) were published by Alves and Kariagosova [7, 8], as well as Langseth [9, 10], who investigated long bars of top hat section and Teramoto et al. [11].

2. Aim and subject of the analysis

The present work is devoted to the strain–rate sensitivity analysis, exactly: to the analysis of strain–rate influence on loading paths of thin–walled sections subjected to dynamic, progressive compression or bending. The loading paths are determined using the analytical solution based on yield line analysis (plastic mechanism approach) and, simultaneously, on the basis of FE numerical simulation.

The analytical solution has been achieved using an approach similar to that applied by Jones [4]. Subjects of the analysis were two cases: top–hat section bar subjected to uniform compression (Fig. 1a) and box–section beam subjected to pure bending (Fig. 1b). Preliminary results for the second case were published in [6].

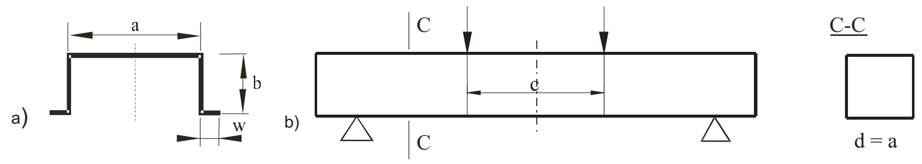


Figure 1 Thin-walled members under investigation; a) top hat-section column subjected to compression, b) – box-section beam under pure bending

3. FE analysis

Numerical FE analysis [12] was performed using commercial software ANSYS version 10 package. In FEM model shell element (SHELL181) was applied. It was four nodes element with six degrees of freedom in each nodes (translations in x, y and z directions of local coordinate system and rotations about these axis, respectively). The element formulation is based on logarithmic strain and true stress measures and it is well suited for nonlinear large strain applications. The element allows to apply different material characteristics, taking into account plastic and visco–plastic effects.. In the analysis the bilinear description was applied. The strain rate sensitivity was achieved by Perzyna [13] model application where the coefficients were chosen in accordance to Cowper–Symonds model [5] – relation (2). The visco–plastic Perzyna model applies the isotropic strain hardening and the integration of constitutive relations is based on repetitive mapping in order to assure a compatibility of the tangent stiffness matrix and the stress matrix at the end of each step of integration.

$$\dot{\varepsilon} = \gamma \left(\frac{\sigma}{\sigma_Y} - 1 \right)^{1/m} \tag{2}$$

Relation (2) corresponds to the Cowper-Symonds relation (1), where $\gamma = D$ and $1/m = q$. The calculations were performed in two steps. First it was eigen-buckling analysis to obtain buckling mode of the member. The first buckling mode was updated on to member geometry as the initial imperfection of its walls. In the second step the principal non–linear analysis was performed. The increasing load was a control parameter in subsequent steps up to the formation of the plastic mechanism in the analyzed structure.

Plastic mechanisms obtained using the FE simulation were similar to the theoretical models applied in the analytical solution.

It has to be mentioned that the effective strain rate was not constant during the whole process of loading.

4. Load–capacity under collapse – analytical solution

Generally, a determination of the load–carrying capacity of a thin–walled structure can be achieved by means of the limit analysis based on the lower and upper bound theorems of the theory of plasticity [15, 16, 17]. The application of these theorems leads to the lower or upper bound estimation of the load–carrying capacity.

As it was mentioned above, the upper–bound estimation of the load–carrying capacity can be achieved by means of the solution based on the plastic mechanism (yield line) analysis.

The plastic mechanism approach is based on two basic methods [3,16], namely the *energy method (work method)* and the *equilibrium strip method*. The energy method is widely used in analyzing collapse behaviour of thin–walled structures, particularly crushing and progressive buckling of thin–walled columns and beams. It is capable to estimate an energy dissipation directly, which is especially important in the analysis of energy absorbers. Furthermore, it enables to evaluate an energy absorbed not only at so called stationary yield lines, but at traveling and rolling yield lines (plastic corners) as well [3, 16].

The equilibrium strip method treats the plastic mechanism as a compatible collection of strips of infinitesimal or unit width parallel to the direction of applied force. On the basis of free–body–diagram of a separated strip an equilibrium equation is formulated and then, those equations are integrated across walls of the plastic mechanism, in order to obtain simultaneous equilibrium equation for the mechanism as a whole [16]. An application of this method is restricted to the analysis of local plastic mechanisms build of stationary yield lines only. It is widely used in investigations of plated columns under compression and delivers a direct relation between an applied compressive force and the deflection of the column.

This method has been applied by Jones [4], who incorporated directly the dynamic flow stress into the relation governing the axial crushing force of a thin–walled tube.

However, this method does not allow to investigate an influence of actual strain–rates different at different yield strips, upon the crushing behaviour of a structure.

In the paper the energy method is applied. Thus, starting from the Principle of Virtual Velocities, work (power) of external generalized forces has been compared with the energy (rate of change) of plastic deformation :

$$P\dot{\delta} + M\dot{\theta} = \sum_i \int_{A_i} (N_0 \dot{\varepsilon}_p) dA_i + \sum_j \dot{E}_{gi}(\dot{\beta}, \bar{m}_{ip}\chi) \tag{3}$$

where:

P, M – generalized forces,

$\dot{\delta}$ and $\dot{\theta}$ – rates of change of generalized displacements (shortening and angle of rotation at the plastic hinge),

β - vector of kinematical parameters of the plastic mechanism,

χ – vector of geometrical parameters of the plastic mechanism,

N_0 – vector of membrane forces per unit length,

$\dot{\varepsilon}_p$ – rate of change of membrane strain.

The algorithm of load–capacity at collapse for the members mentioned above subjected to dynamic load have been derived, taking into account the strain rate.

In the analytical solution of the plastic mechanism problem, based on the rigid–plastic theory, one has to replace the initial yield stress corresponding to the static load with the initial yield stress under dynamic load, which depends on the strain rate. In the present solution the fully plastic moment $\bar{m}_{ip}$ [3, 18] at the ith yield line is expressed in terms of the plastic flow stress $\bar{\sigma}_{0i}$ depending on strain rate $\dot{\varepsilon}_{pi}$ at ith yield line (different at different yieled lines),using the Symonds – Cowper relation:

$$\bar{\sigma}_{0i} = \sigma_Y \left[1 + \left(\frac{\dot{\varepsilon}_{pi}}{D} \right)^{1/q} \right] \tag{4}$$

where:

σ_Y – initial yield stress for static load,

$\bar{\sigma}_{0i}$ – initial, plastic flow stress at ith yield line,

$\dot{\varepsilon}_{pi}$ – strain rate at ith yield line,

q and D – material coefficients.

Additionally, following assumptions were taken into the analysis:

1. The strain– hardening effect was neglected, so that the rigid– perfectly plastic material is taken into consideration. According to the previous works [4], this approximation gives satisfactory results for steel and other metallic alloys.

2. The strain-rate $\dot{\varepsilon}_{pi}$, taken into account in (4) , is of the maximum magnitude, corresponding to the maximum, jamming angle δ_{max} or θ_{max}, when the plastic mechanism is entirely developed and takes the final position, i.e. is jammed,

3. The maximum strain rate $\dot{\varepsilon}_{pi}$is calculated under the assumption of the following postulate (Kotelko – [3,18]), which relates the plastic strain ε_{pi} with the angle of rotation $\beta_i$of two adjacent walls of the plastic mechanism at the i–th yield line:

$$\varepsilon_{pi} = \frac{\beta_i}{2n} \tag{5}$$

where n is a multiple of the beam wall thickness established in the way described by Kotełko [18]. The limit value of membrane strains was calculated from geometrical relations of a plastic mechanism,

1. The input data in the algorithm were mean value of the impact velocity v_0 and geometrical parameters of the plastic mechanism. It was assumed, that the time from the instant of impact to the instant when the mechanism is jammed amounts

$$T = \delta_{l\,\lim}/\nu_0 \qquad \text{or} \qquad T = \theta_{\lim}/\nu_0$$

Thus, using the postulate (5), the maximum value of the strain rate for the ith yield line was calculated, as follows:

$$\dot{\varepsilon}^p_{i\,\max} = \frac{\beta_{i\,\lim}}{T(2n)} \tag{6}$$

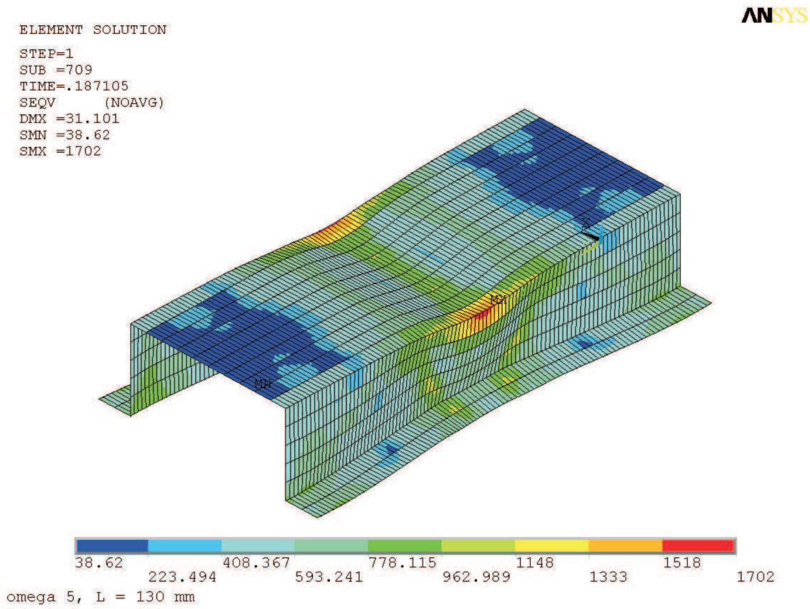
under assumption, that $\dot{\beta}_{i\,\lg r} = \frac{\beta_{i\,\lim}}{T}$. Finally, after taking into account relation (6) the fully plastic moment at the i–th yield line was derived in the following form:

$$\bar{m}_{pi} = \sigma_Y \left[1 + \left(\frac{\dot{\varepsilon}^p_{i\,\max}}{D} \right)^{1/q} \right] \frac{t^2}{4} \tag{7}$$

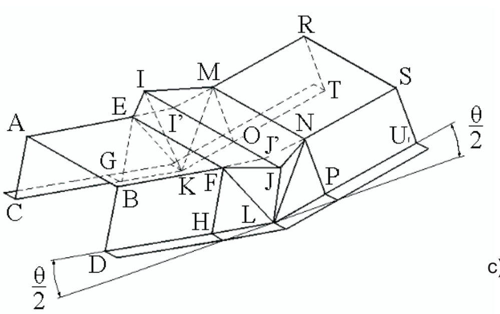
The total energy of plastic deformation (3) was derived in the incremental numerical procedure – for subsequent increments of the rotation angle $\Delta\theta$ or linear shortening $\Delta\delta$ and for corresponding increments of rotation angles $\Delta\beta_I$at particular i–th yield line. Then, an instant moment capacity at the global plastic hinge or a compressive force was calculated in the procedure of numerical derivation.

4.1. *Column under compression*

For the top hat–section column (Fig. 1a) subjected to uniform axial compression, the analytical solution has been based on the theoretical model of the plastic mechanism (Fig. 2b) adequate for columns of shape ratio $b/a \geq 0.67$. The model has been confirmed by FE simulation (Fig. 2a) and also by experimental quasi–static tests (Fig. 2c) – [20]. Using the numerical procedure described above the algorithm of failure curve (unloading path) was derived, which enabled to produce load-shortening diagrams.



a)



b)

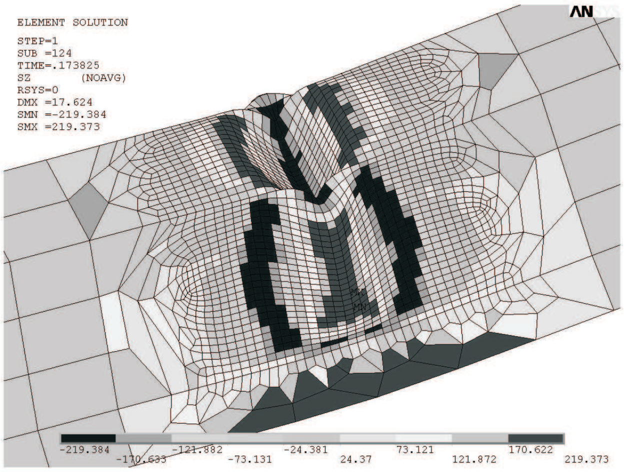
c)

Figure 2 Plastic mechanism of failure of top hat-section column: a) – FE deformation pattern [12], b) – theoretical model [10] ,c) – real mechanism of failure (quasi–static test)

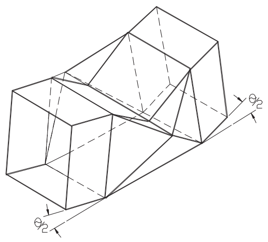
4.2. *Box–section beam under pure bending*

Similarly to the solution for column under compression, the algorithm of failure curve for box–section beam under pure bending was elaborated, which enabled to produce diagrams of bending moment capacity in terms of angle of rotation θ at the global plastic hinge. The solution was based on the theoretical model of plastic mechanism, originally created by Kecman (Fig. 3b), described in details in [3]. This model was verified by both numerical FE simulation (Fig. 3a) and experimental tests [21].

a)



b)



c)

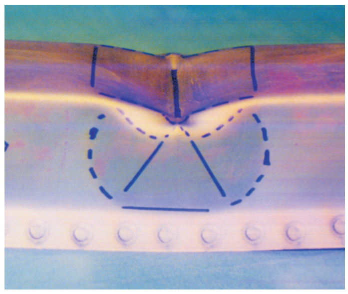


Figure 3 Plastic mechanism of failure in box–section beam under pure bending: a) – FE simulation, b) – theoretical model, c) – global plastic hinge in steel beam (experimental test [21])

5. Comparative analysis of results

Simultaneously with calculations based on the analytical plastic mechanism approach, numerical FE calculations were carried out. In both examined cases (column under compression and beam under pure bending), the FE analysis was performed in the way described in paragraph 3. It should be underlined here, that in FE iterative procedure the effective strain rate varied during the whole process of loading, while in the analytical solution the strain rate at particular yield lines was of constant, maximum value, given by relation (6).

Fig.4. shows diagrams of plastic strain rate in terms of control parameter (shortening or angle of relative rotation at the global plastic hinge, respectively) for both cases under investigation, obtained in the iterative FE procedure [12]. The strain rate was evaluated as an increment of plastic strain for the corresponding time step.

Fig.4a shows the strain rate vs shortening and additionally, equivalent Huber - von Mises stresses in the initial phase of iterative procedure (for shortening up to 2 mm). Fig.4b presents the loading path and corresponding diagram of $\dot{\varepsilon}$ vs angle of rotation,for different material models – bilinear with isotropic hardening (BISO) and Perzyna model for bilinear material taking into account the strain rate effect.

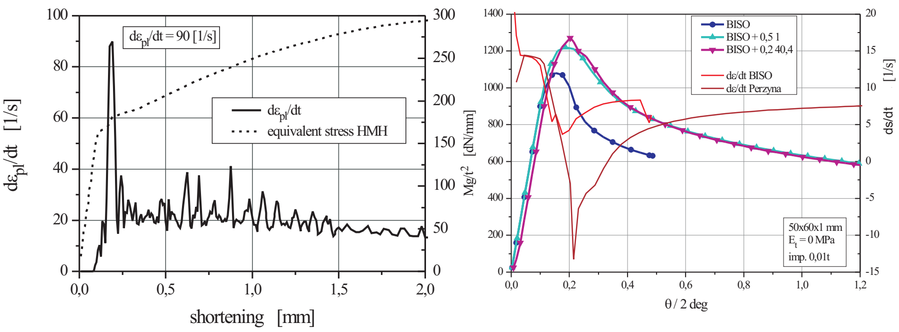


Figure 4 Plastic strain rate in FE iterative procedure: a) column under compression [12], b) – beam under pure bending [6]

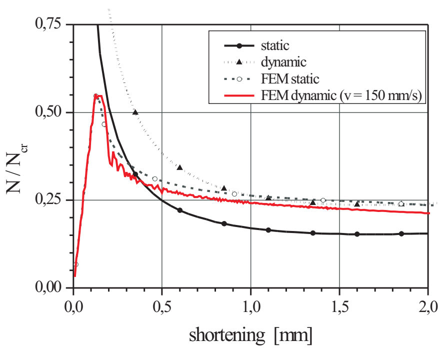


Figure 5 Load–shortening diagrams (theoretical results)

5.1. *Column under compression [12]*

Detailed numerical calculations were carried out for the steel column of parameters given in Tab. 1. In Fig.5 comparative results of numerical calculations are presented. Loading paths obtained from analytical solution and FE simulation are shown both for static and dynamic load at the initial impact velocity $v_0 = 150$ mm/s. Curves described as "static" and "dynamic" concern the analytical solution. Other curves were obtained from FE calculations.

Fig. 6a shows results of analytical solution, based on the plastic mechanism approach. The loading path is presented, together with unloading paths (failure curves) corresponding to different maximum strain rates. These strain rates are maximum values of strain rates obtained at that yield line of the plastic mechanism, at which the strain rate reaches the maximum value.

Table 1 Parameters of top hat–section column

Material parameters	Dimensions
σ_Y= 165 MPa. E = 192000 MPa; E_t = 3000 MPa q = 5; D = 40.4 [s^{-1}]	a= 30 mm b = 30 mm w = 8 mm t = 0.6 mm column length l = 150 mm

5.2. *Beam under pure bending*

Numerical calculations were carried out for steel box–section beam of material parameters and dimensions shown in Tab. 2. In Fig. 6b diagrams of bending moment capacity vs angle of deflection (rotation) are shown. The diagrams represent loading and unloading paths in the whole range of loads consisting of pre– and post–buckling path obtained from analytical solution based on the asymptotic method and effective width approach [3] and post–failure paths. The post–failure path for static load is presented (continuous line) together with three paths corresponding to three different maximum strain rates. The maximum strain rates were determined in the way described above.

Table 2 Parameters of box–section beam

Dimensions	Material parameters
a = 50 mm; b = 60 mm (web height)	E = $2*10^5$ MPa; σ_Y= 190 MPa E_t= 3000 MPa
t = 0.75 ; 1.00; 1.25 [mm]	q = 5; D = 40.4 [s^{-1}]

6. Final remarks

The numerical results confirmed the strong influence of the strain rate of the material on the crushing behaviour of the thin–walled member. The load–carrying capacity of the structure increases due to the increase of the strain rate, as well as the failure curve representing the collapse behaviour is situated above the corresponding curve for static loading. Results of the analytical solution based on the plastic mechanism approach are more sensitive to an increase of the strain rate than results of FE analysis, particularly in the case of column under compression.

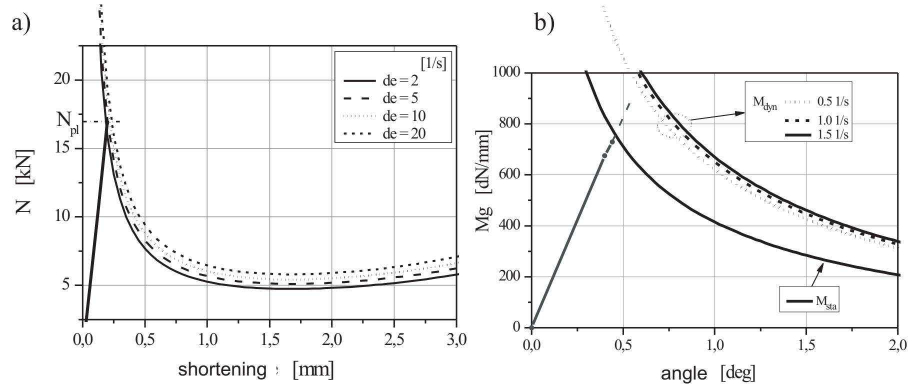


Figure 6 Influence of strain rate upon the unloading paths of thin walled members: a) – column under compression, b) – beam under bending

In that case the agreement between analytical and FE results is good. The discrepancy is higher in the case of beam under pure bending [5].

However, in both cases an agreement of results of FE simulation and plastic mechanism analysis is satisfactory enough, so that the latter can be applied for upper bound estimation of load–carrying capacity, in the way described in the Introduction. This approach leads to a time–saving numerical procedure.

Further research should be continued into the incorporation of the strain–hardening effect in the analytical solution as well as into the more exact analytical solution taking into account the change of the strain rate during crushing process. Also compatibility of FE and analytical results remain an open question because of change of the strain rate in the FE transient dynamic analysis, which is not the case in the analytical solution.

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